We have therefore

$$z = \frac{10 - 3\pi}{6(3 - 1\cdot 8)} . R . \qquad (6)$$

Collecting results and expressing the coefficients by common logarithms we have

$$H = [9.98408] . PR^2, \qquad V = [9.73160] . PR^2,$$
$$\bar{x} = [9.88867] . R, \qquad \bar{x} = [9.80943] . R,$$
$$\bar{z} = [9.55695] . R, \qquad \bar{z} = [9.74724] . R.$$

If we assume $P = 100$, and $R = 22$ feet, we have

$$H = 46658 \text{ pounds}, \qquad V = 26088 \text{ pounds},$$
$$\bar{x} = 17.0 \text{ feet}, \qquad \bar{x} = 14.2 \text{ feet},$$
$$\bar{z} = 7.9 \text{ feet}, \qquad \bar{z} = 12.3 \text{ feet}.$$

[*A. Hall.*]

[The formula quoted in the question is empirical and was derived by Duchemin from experimental results given by Vince, (*Philosophical Transactions of the Royal Society of London*, 1778); by Hutton, (*Resistance of the Air to Bodies in Motion*, Tract 36, 1788); and by Thibault, (*Recherches experimentales sur la resistance de l'air*). Hutton represented the results of his own experiments alone quite exactly by the formula

$$(\cos i)^{1.842 \sin i}.$$

This formula represents the whole group of data however less well than Duchemin's.—Ed.]

EXERCISES.

7

In the theory of perturbations, if the differential equations have the form

$$\frac{d^2\xi}{dt^2} + \frac{k^2(1+m)}{r^3} . \xi = A,$$

prove that

$$k\sqrt{p_0} . \xi = \xi_0 + \xi_1 + \xi_2 + \ldots,$$

where

$$\xi_0 = y_0 \int A x_0 \, dt - x_0 \int A y_0 \, dt + c_1 y_0 - c_2 x_0,$$

$$\xi_{n+1} = y_0 \int B \xi_n x_0 \, dt - x_0 \int B \xi_n y_0 dt,$$

$$B = \frac{k^2 \sqrt{1+m}}{\sqrt{p_0}} \left(\frac{1}{r_0^3} - \frac{1}{r^3} \right),$$

in which c_1 and c_2 are constants of integration and p_0, r_0, x_0, y_0 refer to an assumed elliptic orbit. [*Ormond Stone.*]

8

If an ellipse and a rectangular hyperbola have the same centre, and the hyperbola passes through the focus of the ellipse, then at the point of intersection of the curves the ellipse makes equal angles with the hyperbola and the central radius. [*H. A. Newton.*]

9

It is assumed that when a gate in a water pipe is closing the pressure increases uniformly and the discharge decreases uniformly. Investigate an expression for the shortest safe time for closing the gate on the basis of these hypotheses: given the length of the pipe, the velocity of the stream, the working pressure, and the greatest admissible pressure to which the pipe may be exposed. [*W. M. Thornton.*]

10

Required the length of a thread wrapped spirally round the frustum of a given cone, the distance between the spires along the slant height being constant. [*A. B. Nelson.*]

11

In exercise 4, what is the probability that the circle exceeds the average circle? [*Artemas Martin.*]

12

The result

$$-\frac{p^2q^2 + 4p^3r - 8q^3 + 2pqr + 9r^2}{(r - pq)^2}$$

is given as the equivalent of the function

$$\left(\frac{\beta - \gamma}{\beta + \gamma}\right)^2 + \left(\frac{\gamma - \alpha}{\gamma + \alpha}\right)^2 + \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2,$$

where α, β, γ are the roots of the cubic

$$x^3 + px^2 + qx + r = 0.$$

Is this result correct? [*A. Hall.*]